

Generalized Parton Distributions and Transversity in Nucleons and Nuclei.

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1. INTRODUCTION

Since they were first introduced in the mid '90s [1,2,3] Generalized Partons Distributions (GPDs) have sensibly transformed our view of hadronic structure. In fact, in addition to providing a framework to describe in a partonic language the orbital angular momentum carried by the nucleon's constituents, they also give direct new information on the partonic distribution in the transverse direction with respect to the large longitudinal momentum in the reaction. In [4,5] GPDs were shown to be related by Fourier transformation to the Impact Parameter dependent Parton Distribution Functions (IPPDF), originally defined by Soper [6]. However, GPDs are not directly related to the so-called Unintegrated Parton Distributions (UPDs) appearing *e.g.* in transverse spin polarized reactions, since the transverse coordinate characterizing both GPDs and IPPDFs, is not Fourier conjugate to the intrinsic transverse momentum in a UPD. A relation can instead be established between the UPDs and the non diagonal elements of the GPD (IPPDF) matrix [7].

All of the recent studies connecting coordinate space and momentum space descriptions promise a whole new dimension for studying hadronic structure that has just begun to unravel: GPDs and UPDs are in fact themselves projections of a more comprehensive theoretical quantity describing a seven-dimensional phase space, known as Wigner Distribution (WD) [8].

A number of new efforts to establish a phenomenology that would allow one to interpret experimental measurements in terms of GPDs, UPDs, ... *etc.*, exist (for a review see *e.g.* [5]). In this paper, in particular, we explore the role of GPDs in nuclei, both as a tool to access hadronic configurations' radii (cf. the Color Transparency (CT) hypothesis [9,7,10]), and as a method to study the nature of nuclear medium induced modifications of the quark and gluon structure of hadrons.

2. DEFINITIONS

GPDs are most easily defined as the structure functions that appear in the deeply virtual Compton scattering reaction $ep \rightarrow e'\gamma$ depicted in Fig.1. Two GPDs denoted by H and E , corresponding to the two possibilities for the final particle's helicity, describe the

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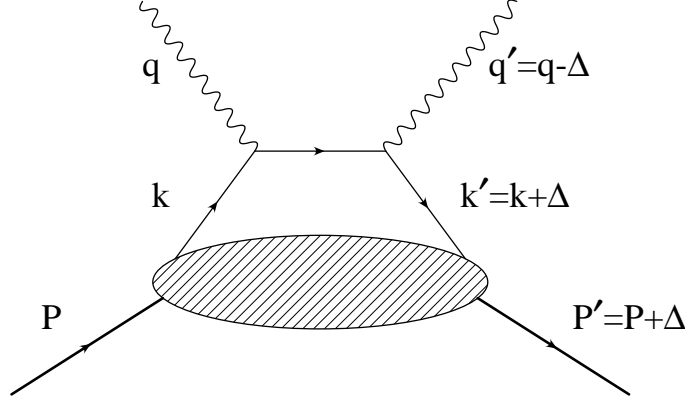


Figure 1. Amplitude for deeply virtual Compton scattering process at leading order in Q^2

process. The kinematical invariants that H and E depend on are defined respectively to a “reference vector”. This can be either $\bar{P} = (P + P')/2$, the average nucleon momentum, or P , the initial nucleon’s momentum. In the first case, one has: $x = (k + k')^+ / (P + P')^+$, $\xi = -\Delta^+ / 2\bar{P}^+$, and $t = -\Delta_\mu^2$ [2]; in the second, $X = k^+ / P^+$, $\zeta = -\Delta^+ / P^+$, and $t = -\Delta_\mu^2$ [3]. The two sets of variables are completely equivalent. The set (X, ζ, t) is, however, best suited both for considering the convolution with nuclear variables, as well as for perturbative QCD evolution.

A relationship was found by Burkardt [4] between GPDs and the IPPDFs – the joint distribution, $dn/dxd\mathbf{b} \equiv q(x, \mathbf{b})$ representing the number of partons of type q with momentum fraction $x = k^+ / P^+$, located at a transverse distance \mathbf{b} (\mathbf{b} is the impact parameter) from the center of P^+ of the system [6]:

$$q(x, \mathbf{b}) = \int \frac{d^2\Delta}{(2\pi)^2} e^{-i\mathbf{b}\cdot\Delta} H_q(x, 0, -\Delta^2) \quad (1)$$

$$H_q(x, 0, -\Delta^2) = \int d^2\mathbf{b} e^{i\mathbf{b}\cdot\Delta} q(x, \mathbf{b}). \quad (2)$$

Since $q(x, \mathbf{b})$ satisfies positivity constraints and it can be interpreted as a probability distribution, $H_q(x, 0, -\Delta^2)$ is also interpreted as a probability distribution, namely the Fourier transformed joint probability distribution of finding a parton q in the proton with longitudinal momentum fraction x , at the transverse position \mathbf{b} , with respect to the center of momentum of the nucleon. The *radius* of the system of partons, which is needed for quantitative CT studies, is [7]:

$$\langle \mathbf{r}^2(x) \rangle^{1/2} = \text{MAX} \left\{ \langle \mathbf{b}^2(x) \rangle^{1/2}, \langle \mathbf{b}^2(x) \rangle^{1/2} \frac{x}{1-x} \right\}. \quad (3)$$

The UPD, $f(x, \mathbf{k})$, is defined as [7]:

$$f(x, \mathbf{k}) = \int d^2\mathbf{b} \int d^2\mathbf{b}' e^{i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} q(x, \mathbf{b}, \mathbf{b}'), \quad (4)$$

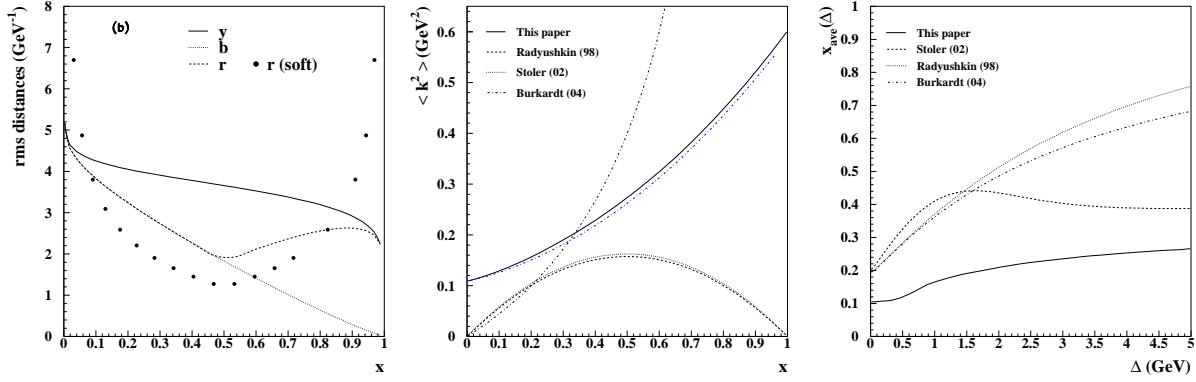


Figure 2. The hadronic configuration's radius, Eqs.(3) and (5), (left); the intrinsic transverse momentum, Eqs.(6) and (4), (center); and the average value of x , as a function of $\Delta = \sqrt{-t}$, Eq.(7) (right) (adapted from [7]).

where $q(x, \mathbf{b}, \mathbf{b}')$ is the non-diagonal IPPDF, namely $q(x, \mathbf{b}, \mathbf{b}') \rightarrow q(x, \mathbf{b})$ for $\mathbf{b} \rightarrow \mathbf{b}'$.

Despite $q(x, \mathbf{b})$ is not directly related to $f(x, \mathbf{k})$, \mathbf{b} and \mathbf{k} not being Fourier conjugates of one another, one can describe the behavior of: *i*) $\langle \mathbf{r}^2(x) \rangle^{1/2}$, which is written in term of, $\langle \mathbf{b}^2(x) \rangle^{1/2}$; *ii*) the average intrinsic transverse momentum, $\langle \mathbf{k}(x) \rangle^{1/2}$; *iii*) the average value of x , $\langle x(\Delta) \rangle$, by using a consistent set of nucleon vertex functions to model the soft part of Fig.1. One obtains:

$$\langle \mathbf{b}^2(x) \rangle = \mathcal{N}_b \int d^2\mathbf{b} q(x, \mathbf{b}) \mathbf{b}^2, \quad (5)$$

$$\langle \mathbf{k}^2(x) \rangle = \mathcal{N}_k \int d^2\mathbf{k} f(x, \mathbf{k}) \mathbf{k}^2, \quad (6)$$

$$\langle x(\Delta) \rangle = \mathcal{N}_x \int_0^1 dx x H(x, \Delta) \quad (7)$$

where \mathcal{N}_b , \mathcal{N}_k and \mathcal{N}_x are normalization factors, and $\Delta = \sqrt{-t}$. The quantities in Eqs.(5)-(7) are displayed in Fig.2.

3. PROBING THE TRANSVERSE STRUCTURE OF BOUND NUCLEONS

Nuclei have since long been suggested as “laboratories” to observe several aspects of quarks and gluons dynamics. One can for instance detect small size hadronic configurations by studying the passage of hadrons through nuclear matter, and the conditions for the onset of Color Transparency in exclusive reactions of the type $eA \rightarrow e'p(A-1)$, or $p(\pi)A \rightarrow p'(\pi')p(A-1)$, $\gamma A \rightarrow \pi N(A-1)$. One can also learn about modifications of the confinement size of nucleons embedded in the nuclear medium through inclusive deep inelastic experiments (the so-called EMC effect).

Both exclusive and inclusive types of reactions involve in their description transverse degrees of freedom of the hadrons. In what follows we briefly describe this particular aspect, and we show with a few examples, the prominent role and new insight provided by GPDs.

3.1. Color Transparency

In the hard scattering approach to QCD, exclusive reactions, and similarly inclusive reactions at $x_{Bj} \approx 1$ (where $x_{Bj} = Q^2/2M\nu$, Q^2 and ν being the four-momentum transfer squared and the energy transfer, respectively) are expected to be dominated by hadronic configurations with the minimum number of quarks (anti-quarks), located within a small relative transverse distance, $\approx 1/\sqrt{Q^2}$, (see *e.g.* [11] and references therein). However, lacking any direct experimental proof, it is also possible to envisage situations where the transverse size of exclusive hard processes might not be small, as *e.g.* in [12], due to the persistence of large endpoint contributions to the hadron's wave function.

Small distances can in principle be filtered by studying either the Q^2 dependence of nuclear cross sections, or the dependence on the atomic number, A , at finite (moderate) Q^2 . Large separations are in fact expected to gradually be blocked by the strong interactions occurring in the nucleus since the cross section for hadronic rescatterings is proportional to the hadrons transverse size, as dictated by the Chew-Low-Nussinov mechanism (see [9] and references therein). From a practical point of view, however, current searches for CT might appear to be in a stall as all experiments performed so far seem not to show on one side, any marked trend for the onset of this phenomenon, and, on the other, the observables do not allow one to discern what factors (*i.e.* features of the hadronic interactions, or of the hadronic wave function, or else...) are responsible for any lack of CT.

Generalized Parton Distributions (GPDs) seem to provide the best candidates to explore the existence and observability of small size hadronic configurations. For illustration, in Ref.[7] we considered the $(e, e'p)$ process from a nuclear target where we introduced a nuclear filter for the large transverse size components as follows:

$$\Pi(b) = \begin{cases} 1 & b < b_{max}(A) \\ 0 & b \geq b_{max}(A) \end{cases}, \quad (8)$$

$b_{max}(A)$ being the size of the filter. The transparency ratio is then defined as:

$$T_A(Q^2) = \frac{\left[\int_0^1 dx H_A(x, t) \right]^2}{\left[\int_0^1 dx H(x, t) \right]^2} = \frac{\left[\int_0^1 dx \int_0^{b_{max}(A)} db b q(x, b) J_0(b\sqrt{t}) \right]^2}{\left[\int_0^1 dx H(x, t) \right]^2} \quad (9)$$

By varying the parameter b_{max} , and by using different parametrizations of $q(x, b)$, one can in principle disentangle the effect of the hadronic size from the effect of the hadronic interactions in the nuclear medium.

3.2. Nuclear Deep Inelastic Scattering

GPDs provide also a unique tool to describe the spatial distribution of quarks and gluons in nuclei. Throughout the years since the first discovery of the EMC effect [13], an increasingly coherent picture has emerged of Deep Inelastic Scattering (DIS) processes from nuclei. The main outcome is that nucleons, despite the high resolution achieved in DIS experiment, do not behave as free. Their interactions are instead important, and they are responsible for the modifications of the nuclear cross section with respect to the free nucleon one. Despite the general consensus on this picture, the way these interactions proceed is still largely model dependent, ranging from increasingly sophisticated binding models [14], effective theories [15,16], and “rescaling” of the scale dependence of the effect.

In this paper we use an approach where we account for final state interactions between the outgoing nucleon and nuclear debris, parametrized as off-shell effects.² An important aspect of our approach is that it provides a description of the EMC effect that, at variance with the naive (on-mass-shell) binding models, can simultaneously reproduce both the x_{Bj} and A dependences of the data. We present results for a spin 0 nucleus, namely ${}^4\text{He}$ (more details and evaluations for larger nuclei can be found in [17]). The GPD, H_A , reads:

$$H^A(X, 0, t) = \int \frac{d^2 P_\perp dZ}{2(2\pi)^3} \rho_A(P, P') H^{off,N}(X_N, 0, P^2, t), \quad (10)$$

where we used the (X, ζ, t) set of variables. Moreover, in a nucleus $X = k^+/(P_A^+/A)$, $Z = P^+/(P_A^+/A)$, and $X_N = X/Z \equiv k^+/P^+$, k , P , P_A being the active quark, nucleon, and nuclear momentum, respectively. For an off-shell nucleon, and for $\zeta = 0$, $H^{off,N}$ is defined as:

$$H^{off,N}(X_N, 0, P^2, t) = \frac{X_N}{1 - X_N} \int \frac{dk_\perp^2}{2\pi} \rho_N(k(P), k'(P)), \quad (11)$$

$\rho_A(P, P')$ and $\rho_N(k(P), k'(P))$ are *off-diagonal* nuclear and nucleon spectral functions, respectively. Notice that $H^{off,N}(X_N, 0, P^2, t)$ is modified both kinematically and dynamically with respect to the free nucleon GPD. Kinematical modifications due to Fermi motion and nuclear binding produce a shift in the X dependence with respect to the free nucleon. $H^{off,N}(X_N, 0, P^2, t)$ is however also structurally different from the on-shell case.

Off-shell modifications, differently from Fermi motion and binding, affect the transverse variables. It is therefore of the outmost importance to evaluate carefully their impact on GPDs, especially in view of the fact that these are Fourier transforms of IPPDFs. GPDs in fact, provide a handle to directly evaluate the spatial modifications of the nucleon inside the nuclear medium. By making the assumption that the struck nucleon is located at the center of the nucleus, *i.e.* the nucleon impact parameter (β) distribution is described by: $\tilde{\rho}_A(Z, \beta) \approx \rho_A(Z, P_\perp) \times \delta(\beta)$, we obtain for a bound nucleon

$$\langle b_N^2(x) \rangle_{Bound} = \frac{\int d^2 \mathbf{P}_\perp \int_X^A dZ [\int d^2 \mathbf{b} q(X/Z, \mathbf{b}) \mathbf{b}^2] \rho_A(Z, P_\perp^2)}{\int d^2 \mathbf{P}_\perp \int_X^A dZ f_N^{OFF}(X/Z, P_\perp^2) \rho_A(Z, P^2)}, \quad (12)$$

where $f_N^{OFF}(X/Z, P_\perp^2)$ is the PDF in an off-shell nucleon [14].

In Fig. 3 we present results for: *i*) the average impact parameter squared in a bound nucleon, Eq.(12); *ii*) the intrinsic transverse momentum in a nucleus, calculated by convoluting Eq.(6) with $\rho_A(Z, P_\perp^2)$; *iii*) the ratio $R = [H_A(X, t)/F^A(t)]/[H_N(X, t)/F_1(t)]$, F_A being the form factor for ${}^4\text{He}$ and F_1 being the integral of Eq.(2). All results are part of a preliminary study accounting for the effect of binding and Fermi motion. We find that the main nuclear effect on $\langle b^2 \rangle$ is an enhancement at large x due to Fermi motion. Similarly, the average intrinsic k_\perp is enhanced, as a function of x due to the combined effect of both longitudinal and transverse motion inside the nucleus. Finally, the impact of nuclear effects on GPDs is best studied by normalizing $H_{A,N}$ to the corresponding form factors. We find that both the effect of binding (dip at intermediate X), and of Fermi motion are enhanced at $t \neq 0$. The effects of Fermi motion are in fact sizable at $x \approx 0.6$, a region more accessible experimentally.

²The four-momentum squared of a nucleon inside the nucleus is different from its mass squared, $P^2 \neq M^2$, and it is instead related to the nucleons transverse momentum, P_T .

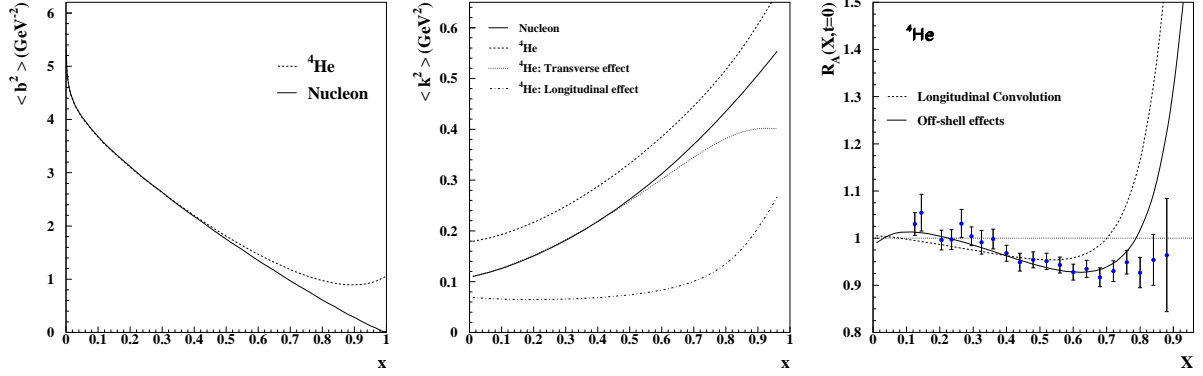


Figure 3. The quark's radius in a nucleon inside ${}^4\text{He}$ (left); the average intrinsic transverse momentum in a nucleon inside ${}^4\text{He}$ (center); and the ratio $R = [H_A(X, t)/F^A(t)]/[H_N(X, t)/F_1(t)]$ of GPDs in ${}^4\text{He}$ to the free nucleon one, normalized by the corresponding form factors (right).

In conclusion our approach for studying nuclear effects in both hard exclusive and inclusive processes using GPDs will allow for a more detailed understanding of the “intrinsic” transverse components in nuclei. In particular, it will be possible to determine whether nucleons’ deformations in the nuclear medium are at the origin of EMC effect.

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